

# Instrumental variables I

## Session 11

PMP 8521: Program evaluation  
Andrew Young School of Policy Studies

# Plan for today

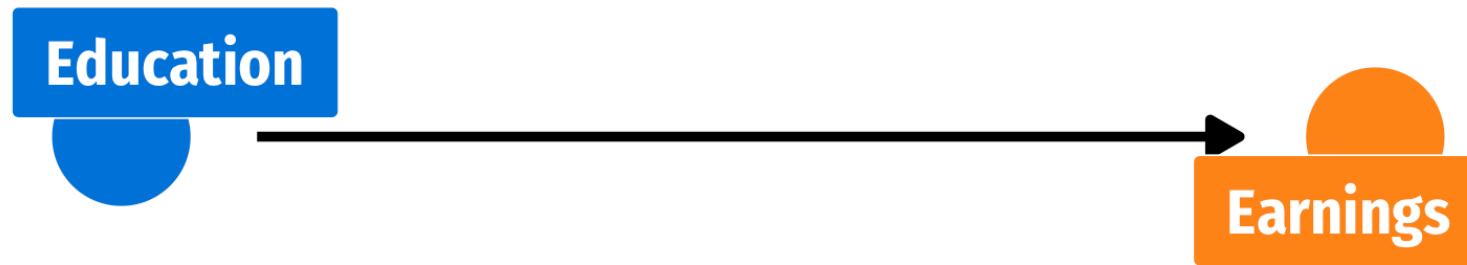
**Endogeneity and exogeneity**

**Instruments**

**Using instruments**

# Endogeneity and exogeneity

# Does education cause higher earnings?



$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

If we ran this regression, would  $\beta_1$  give us the causal effect of education?

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

**No!**

Omitted variable bias!

Unclosed backdoors!

Endogeneity!

# Exogeneity and endogeneity

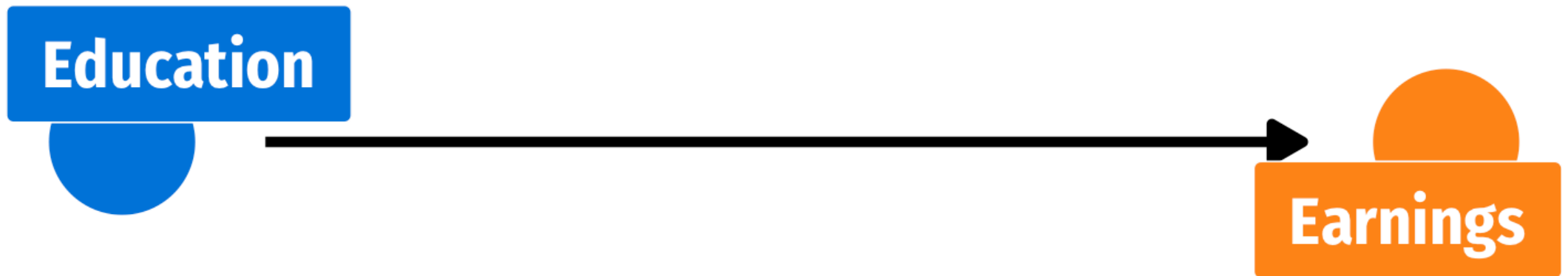
## **Exogenous variables**

**Value is not determined by anything else in the model**

**In a DAG, a node that doesn't have arrows coming into it**

# Exogeneity

Education is exogenous: no arrows *into* it



# Exogeneity and endogeneity

## Endogenous variables

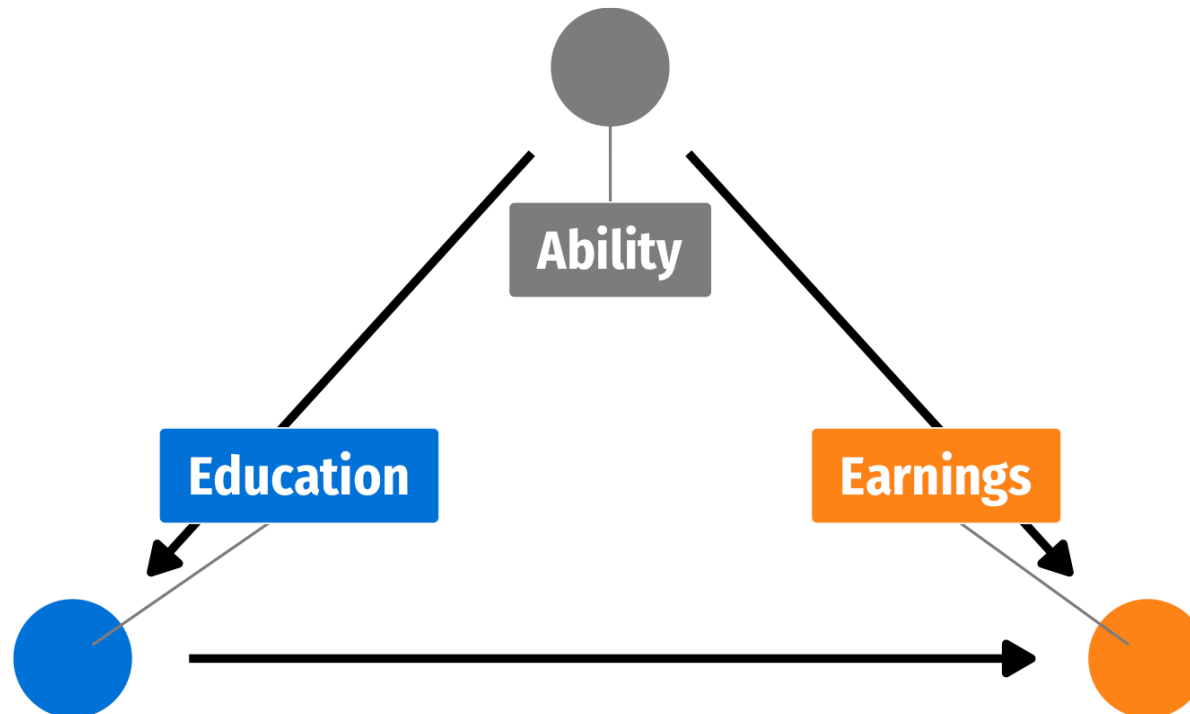
Value is determined by  
something else in the model

In a DAG, a node that  
has arrows coming into it



# Endogeneity

Education is endogenous: Ability  $\rightarrow$  Education

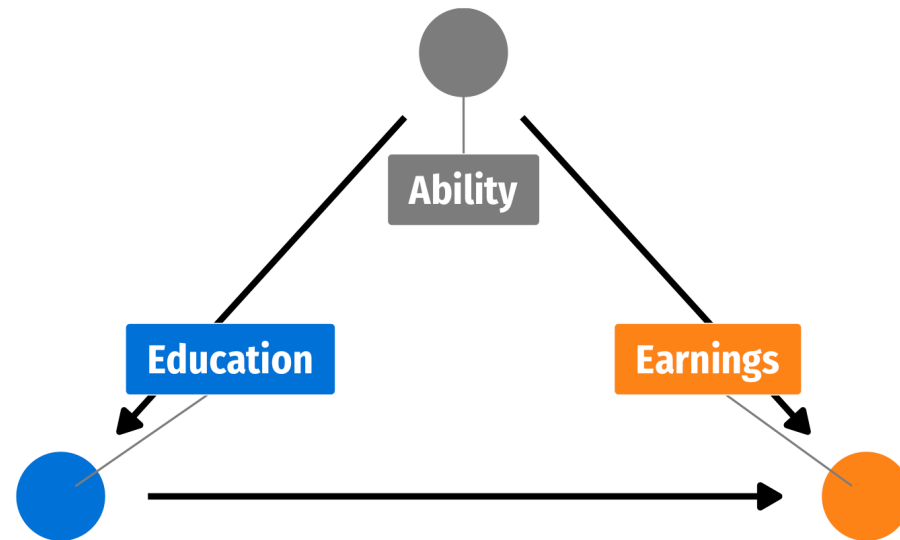


# Exogeneity

**What would exogenous variation in education look like?**

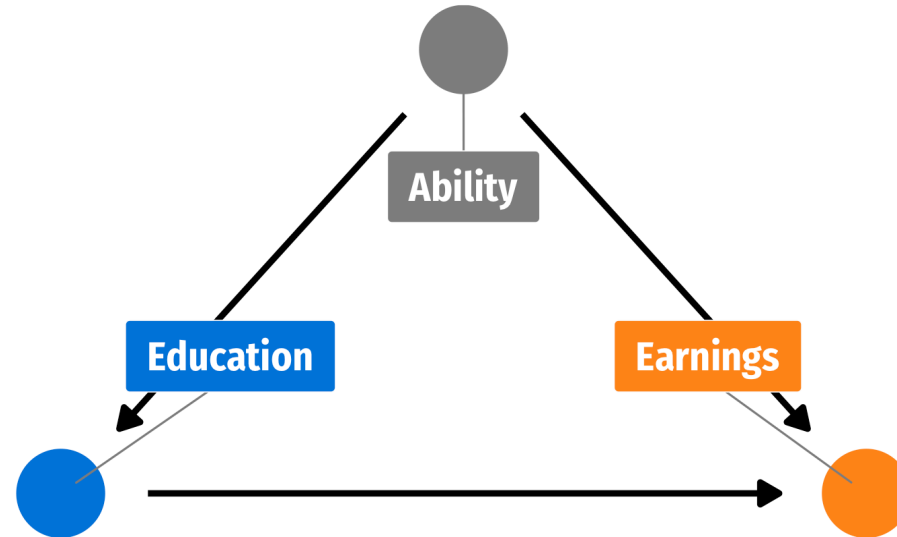
**Choices to get more education that are essentially random (or at least uncorrelated with omitted variables)**

**We'd like education to be exogenous  
(an outside decision or intervention), but it's not!**



**Part of it is exogenous, but part of it is  
caused by ability, which is in the DAG**

# Fixing endogeneity with DAGs



**Close backdoor and adjust for ability**

Adjustment filters out the endogenous part of education and leaves us with just the endogenous part

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \beta_2 \text{Ability}_i + \varepsilon_i$$

|   | <b>Outcome = wage</b> |                 |
|---|-----------------------|-----------------|
|   | <b>Unadjusted</b>     | <b>Adjusted</b> |
| (Intercept)                                       | -59.378***            | -85.571***      |
|   | (10.376)              | (7.198)         |
| educ  | 13.124***             | 7.767***        |
|   | (0.618)               | (0.456)         |
| ability   |                       | 0.344***        |
|   |                       | (0.010)         |
| Num.Obs.  | 1000                  | 1000            |
| R2  | 0.311                 | 0.673           |
| RMSE  | 39.13                 | 26.97           |
| + p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001 |                       |                 |

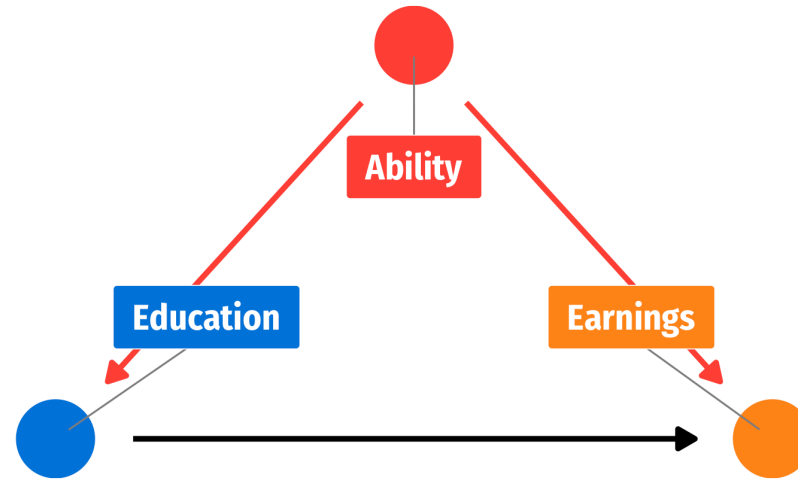
**Unadjusted  
is wrong!**

**Adjusted  
is right!**

**One year of education  
causes hourly wage to  
increase by \$7.77**

**(FAKE DATA)**

# But we can't measure ability!



$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \beta_2 \text{Ability}_i + \varepsilon_i$$

Unmeasurable ability node is in the error term ( $\varepsilon$ )

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

# Split exogeneity and endogeneity

What if we could somehow separate education into its endogenous and exogenous parts?

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

$$\beta_0 + \beta_1 (\text{Education}_i^{\text{exog.}} + \text{Education}_i^{\text{endog.}}) + \varepsilon_i$$

$$\beta_0 + \beta_1 \text{Education}_i^{\text{exog.}} + \underbrace{\beta_1 \text{Education}_i^{\text{endog.}}}_{\omega_i} + \varepsilon_i$$

$$\beta_0 + \beta_1 \text{Education}_i^{\text{exog.}} + \omega_i$$

# Find exogeneity with One Weird Trick™

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i^{\text{exog.}} + \omega_i$$

How do we find only Education<sup>exog.</sup>?

**Use an instrument!**



# Instruments

# What is an instrument?

Something that is correlated with the policy variable

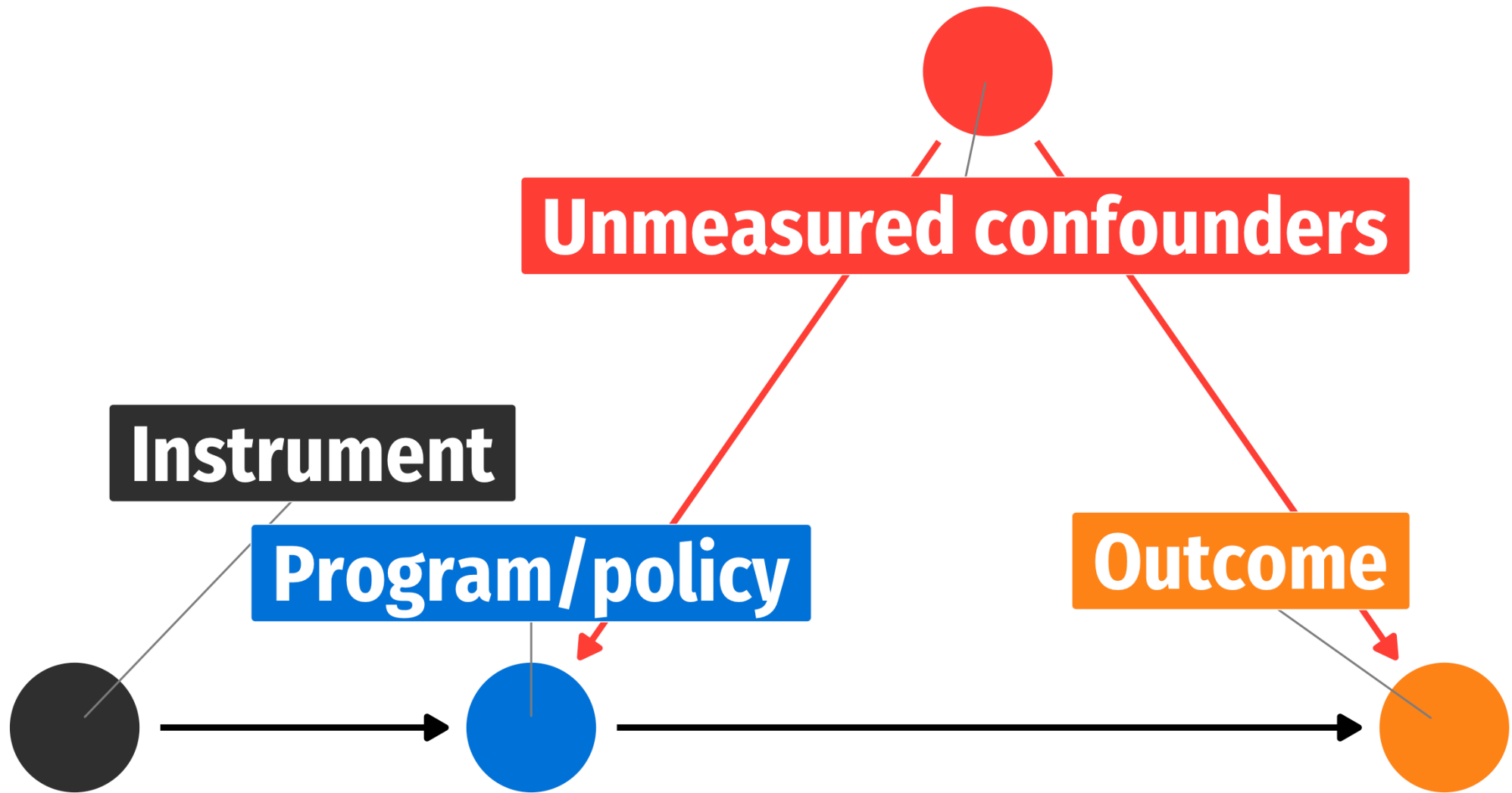
(Relevance)

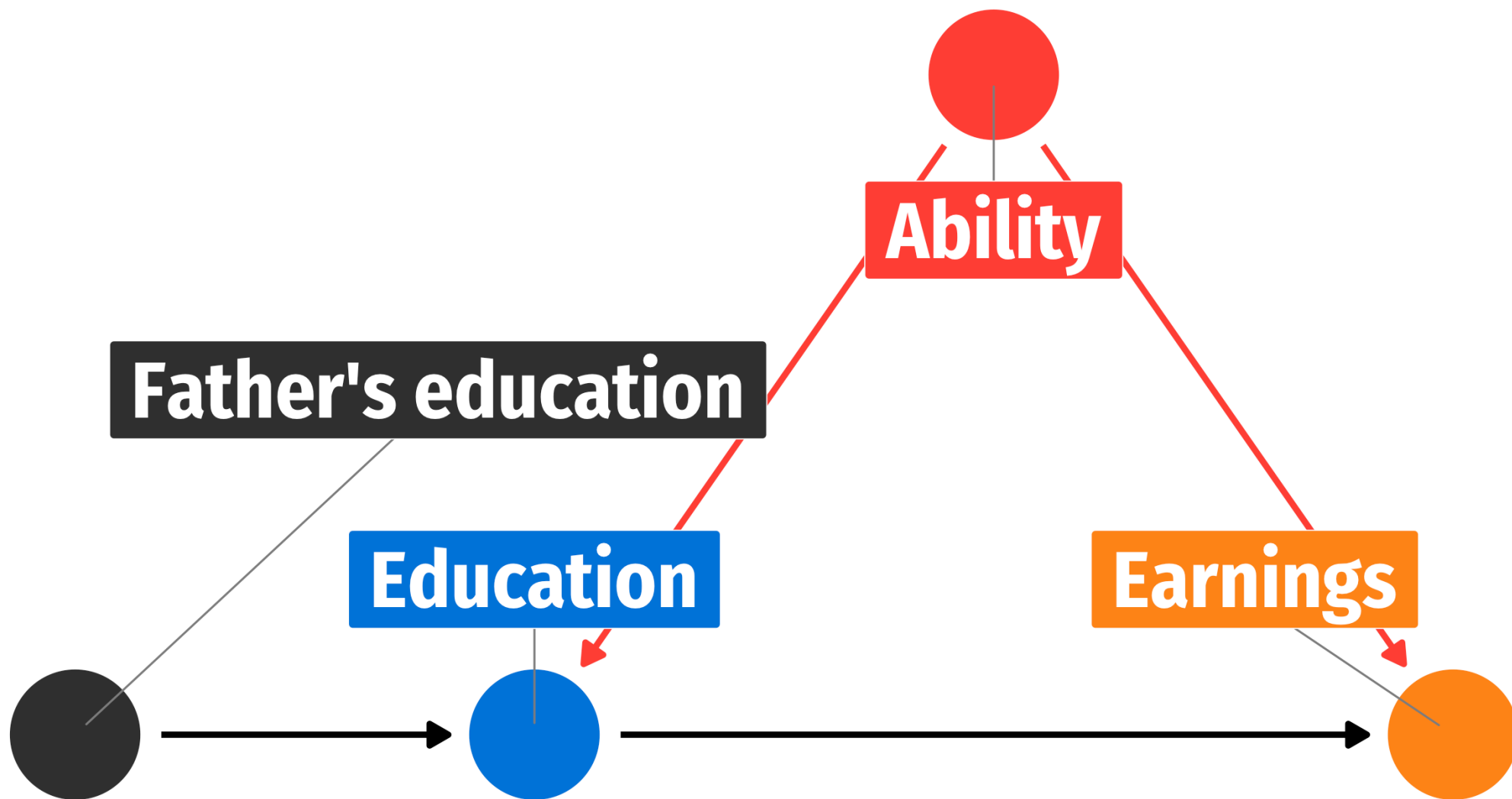
Something that does not directly cause the outcome

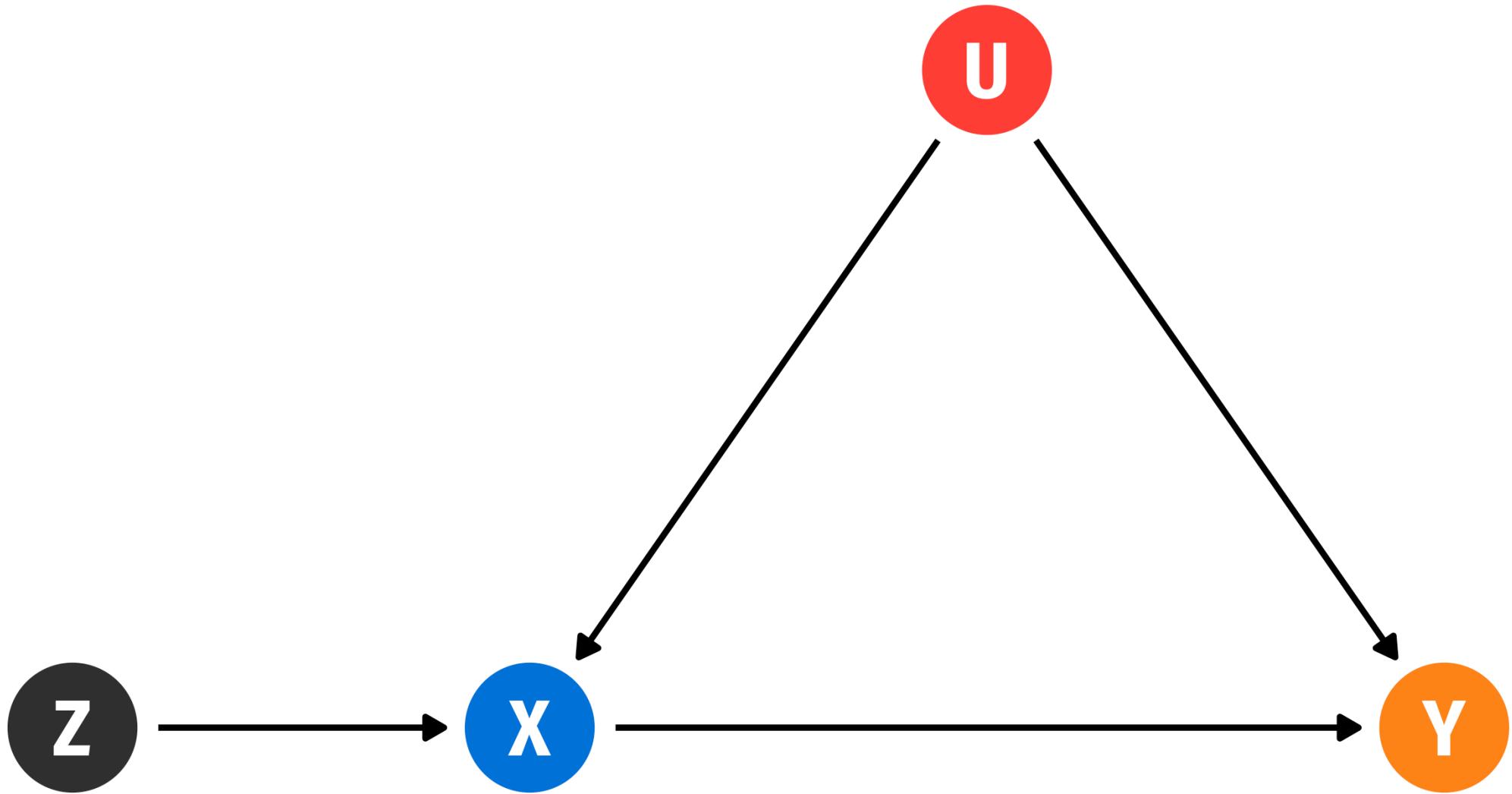
(Exclusion)

Something that is not correlated with the omitted variables

(Exogeneity)







**Relevance**  
Correlated with policy

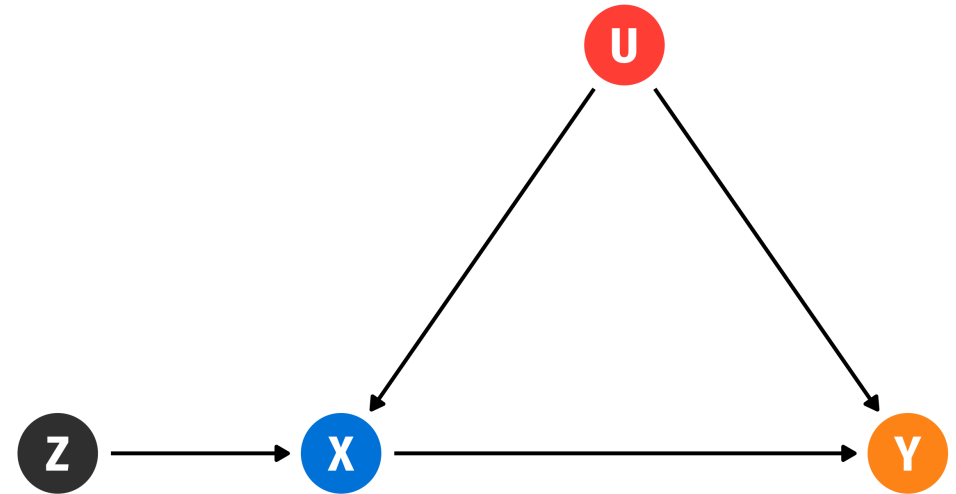
$$Z \rightarrow X \quad \text{Cor}(Z, X) \neq 0$$

**Excludability**  
Correlated with outcome  
*only through policy*

$$Z \rightarrow X \rightarrow Y \quad Z \nrightarrow Y \quad \text{Cor}(Z, Y | X) = 0$$

**Exogeneity**  
Not correlated  
with omitted variables

$$U \nrightarrow Z \quad \text{Cor}(Z, U) = 0$$



Relevance testable with stats

Excludability testable with stats + story

Exogeneity requires story, no stats

# Relevance

**Instrument causes change in policy**

$$Z \rightarrow X \quad \text{Cor}(Z, X) \neq 0$$

**Social security number**

**Probably not relevant (uncorrelated with education)**

**3rd grade test scores**

**Potentially relevant (early grades cause more education)**

**Father's education**

**Relevant (Educated parents cause more education)**

# Excludability

Instrument causes outcome *only through* policy

$$Z \rightarrow X \rightarrow Y \quad Z \not\rightarrow Y \quad \text{Cor}(Z, Y \mid X) = 0$$

Social security number

Exclusive (SSN isn't correlated with hourly wages)

3rd grade test scores

Potentially exclusive (early grades probably don't cause wages)

Father's education

Exclusive (Parent's education doesn't cause your wages (lol))



# Exogeneity

Instrument not correlated with omitted variables

$$U \nleftrightarrow Z \quad \text{Cor}(Z, U) = 0$$

Social security number

Exogenous (Unrelated to anything related to education)

3rd grade test scores

Not exogenous (Grades correlated with other education factors)

Father's education

Exogenous (Birth to parents is random)

# The huh? factor

"A necessary but not a sufficient condition for having an instrument that can satisfy the exclusion restriction is if people are confused when you tell them about the instrument's relationship to the outcome."

Scott Cunningham, *Causal Inference: The Mixtape*, p. 123

| <b>Outcome</b> | <b>Policy</b> | <b>Unobserved stuff</b> | <b>Instrument</b>  |
|----------------|---------------|-------------------------|--------------------|
| Income         | Education     | Ability                 | Father's education |

| Outcome | Policy    | Unobserved stuff | Instrument          |
|---------|-----------|------------------|---------------------|
| Income  | Education | Ability          | Father's education  |
| Income  | Education | Ability          | Distance to college |

| <b>Outcome</b> | <b>Policy</b> | <b>Unobserved stuff</b> | <b>Instrument</b>   |
|----------------|---------------|-------------------------|---------------------|
| Income         | Education     | Ability                 | Father's education  |
| Income         | Education     | Ability                 | Distance to college |
| Income         | Education     | Ability                 | Military draft      |

| <b>Outcome</b> | <b>Policy</b>      | <b>Unobserved stuff</b>         | <b>Instrument</b>   |
|----------------|--------------------|---------------------------------|---------------------|
| Income         | Education          | Ability                         | Father's education  |
| Income         | Education          | Ability                         | Distance to college |
| Income         | Education          | Ability                         | Military draft      |
| Health         | Smoking cigarettes | Other negative health behaviors | Tobacco taxes       |

| <b>Outcome</b> | <b>Policy</b>      | <b>Unobserved stuff</b>         | <b>Instrument</b>   |
|----------------|--------------------|---------------------------------|---------------------|
| Income         | Education          | Ability                         | Father's education  |
| Income         | Education          | Ability                         | Distance to college |
| Income         | Education          | Ability                         | Military draft      |
| Health         | Smoking cigarettes | Other negative health behaviors | Tobacco taxes       |
| Crime rate     | Patrol hours       | # of criminals                  | Election cycles     |

| <b>Outcome</b> | <b>Policy</b>      | <b>Unobserved stuff</b>         | <b>Instrument</b>        |
|----------------|--------------------|---------------------------------|--------------------------|
| Income         | Education          | Ability                         | Father's education       |
| Income         | Education          | Ability                         | Distance to college      |
| Income         | Education          | Ability                         | Military draft           |
| Health         | Smoking cigarettes | Other negative health behaviors | Tobacco taxes            |
| Crime rate     | Patrol hours       | # of criminals                  | Election cycles          |
| Crime          | Incarceration rate | Simultaneous causality          | Overcrowding litigations |



| <b>Outcome</b>       | <b>Policy</b>      | <b>Unobserved stuff</b>         | <b>Instrument</b>        |
|----------------------|--------------------|---------------------------------|--------------------------|
| Income               | Education          | Ability                         | Father's education       |
| Income               | Education          | Ability                         | Distance to college      |
| Income               | Education          | Ability                         | Military draft           |
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| Crime                | Incarceration rate | Simultaneous causality          | Overcrowding litigations |
| Labor market success | Americanization    | Ability                         | Scrabble score of name   |

| <b>Outcome</b>       | <b>Policy</b>      | <b>Unobserved stuff</b>         | <b>Instrument</b>        |
|----------------------|--------------------|---------------------------------|--------------------------|
| Income               | Education          | Ability                         | Father's education       |
| Income               | Education          | Ability                         | Distance to college      |
| Income               | Education          | Ability                         | Military draft           |
| Health               | Smoking cigarettes | Other negative health behaviors | Tobacco taxes            |
| Crime rate           | Patrol hours       | # of criminals                  | Election cycles          |
| Crime                | Incarceration rate | Simultaneous causality          | Overcrowding litigations |
| Labor market success | Americanization    | Ability                         | Scrabble score of name   |
| Conflicts            | Economic growth    | Simultaneous causality          | Rainfall                 |

# Instruments are hard to find!

The trickiest thing to prove is  
the exclusion restriction

Instrument causes the outcome *only through* the policy

Most proposed instruments fail this!

# Rainfall as an instrument

People love using weather as an instrument... buuuuut...

Rain, Rain, Go away: 137 potential exclusion-restriction violations for studies using weather as an instrumental variable

Jonathan Mellon (University of Manchester)

20-10-2020

## Abstract

Instrumental variable (IV) analysis assumes that the instrument only affects the dependent variable via its relationship with the independent variable. Other possible causal routes from the IV to the dependent variable are exclusion-restriction violations and make the instrument invalid. Weather has been widely used as an instrumental variable in social science to predict many different variables. The use of weather to instrument different independent variables represents strong prima facie evidence of exclusion violations for all studies using weather as an IV. A review of 185 social science studies reveals 137 variables which have been linked to weather, all of which represent potential exclusion violations. I conclude with practical steps for systematically reviewing existing literature to identify possible exclusion violations when using IV designs.



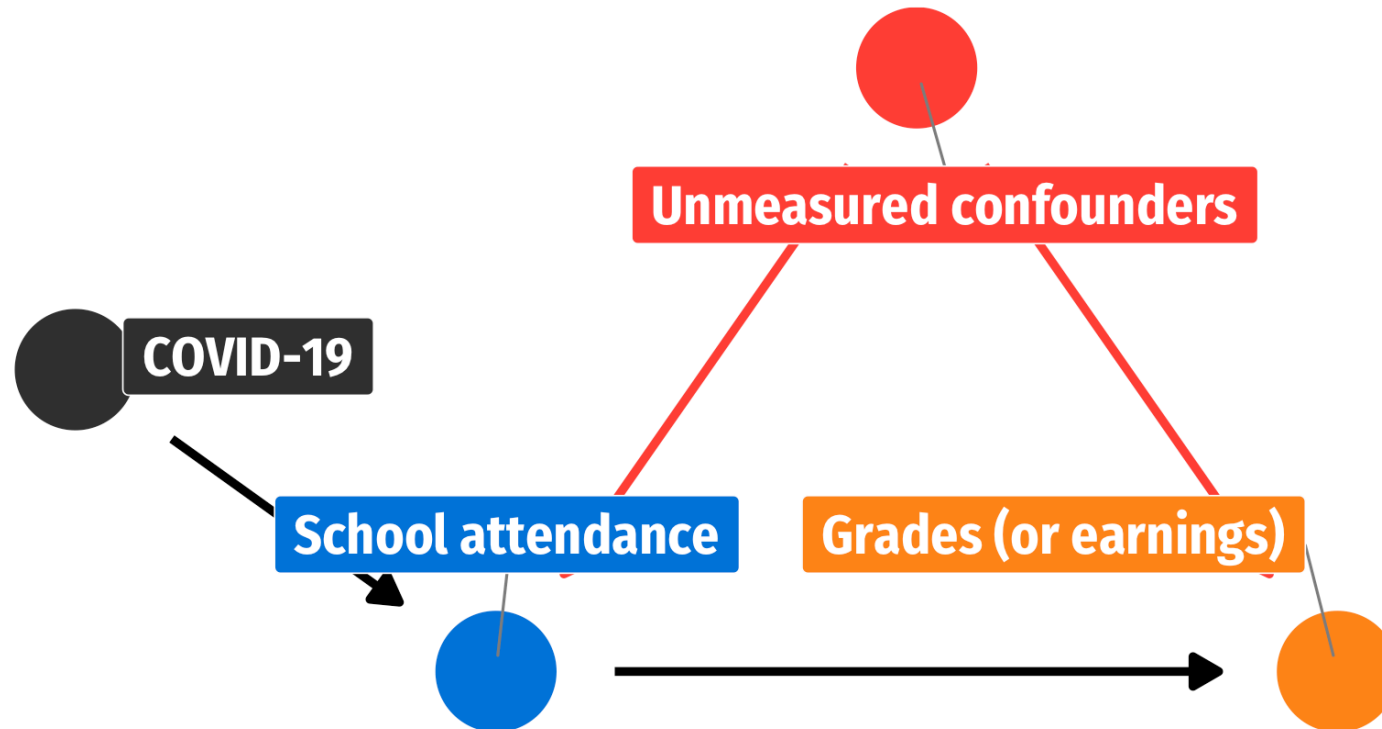
# COVID-19 as an instrument

**A global pandemic is a huge  
exogenous shock to  
social systems everywhere**

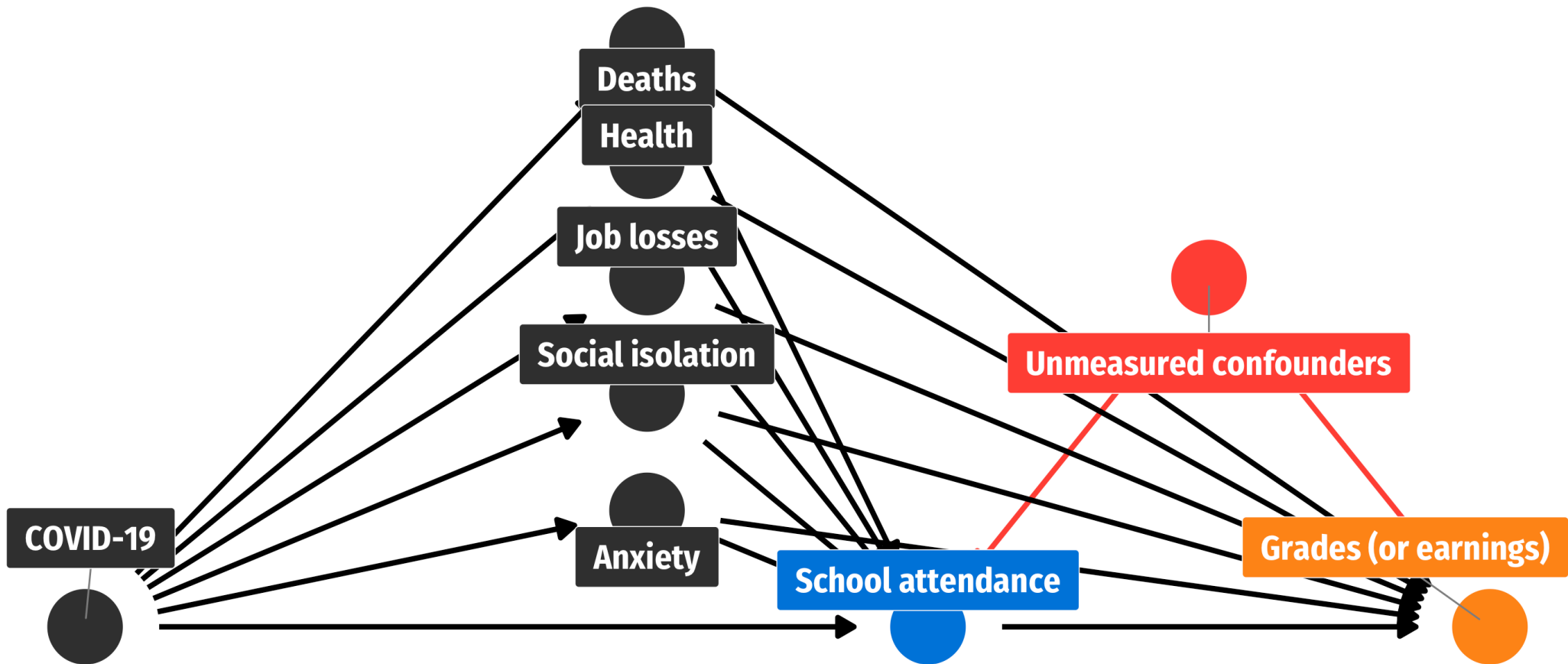
**Maybe we can use it as an instrument!**

# COVID-19 as an instrument

What effect does closing schools have on student performance or lifetime earnings?



# lolnope

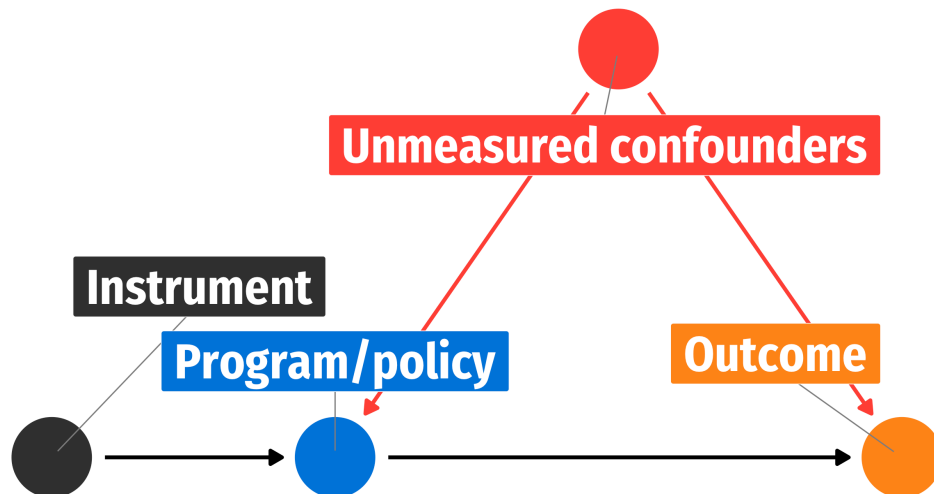




# Falsifying exclusion assumptions

Can you think of some other way that the instrument can cause the outcome outside of the policy?

If so, the instrument doesn't meet exclusion restriction



Instrument → ?? → outcome?

Rainfall → ?? → civil war?

Tobacco taxes → ?? → health?

Scrabble score → ?? →  
Labor market success?

# Using instruments

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

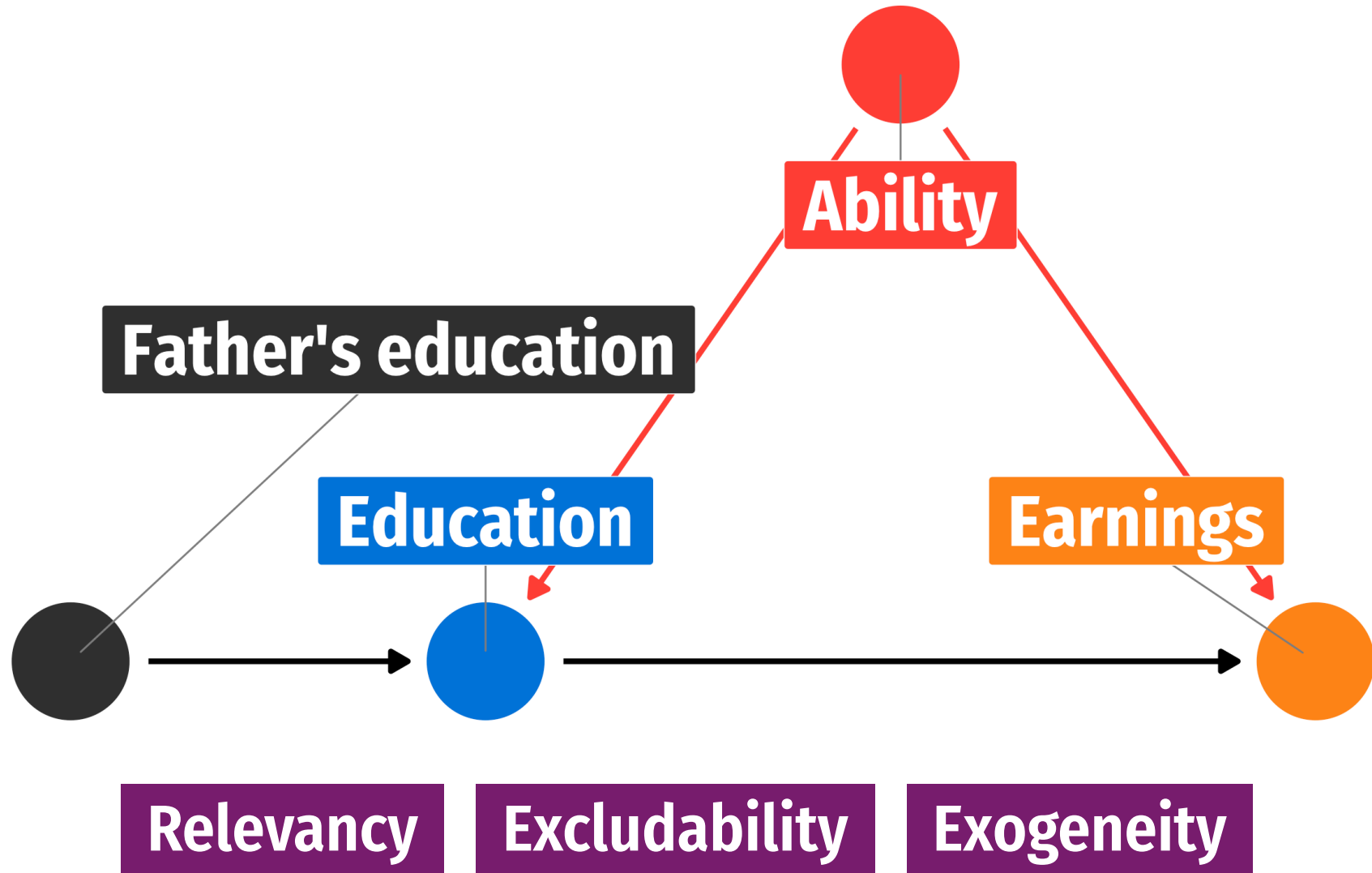
|   | <b>Unadjusted</b> | <b>Forbidden</b> |
|---|-------------------|------------------|
| (Intercept)                                       | -59.378***        | -85.571***       |
|   | (10.376)          | (7.198)          |
| educ  | 13.124***         | 7.767***         |
|   | (0.618)           | (0.456)          |
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| Num.Obs.  | 1000              | 1000             |
| R2  | 0.311             | 0.673            |
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| + p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001 |                   |                  |

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Education}_i + \varepsilon_i$$

$$\beta_0 + \beta_1 (\text{Education}_i^{\text{exog.}} + \text{Education}_i^{\text{endog.}}) + \varepsilon_i$$

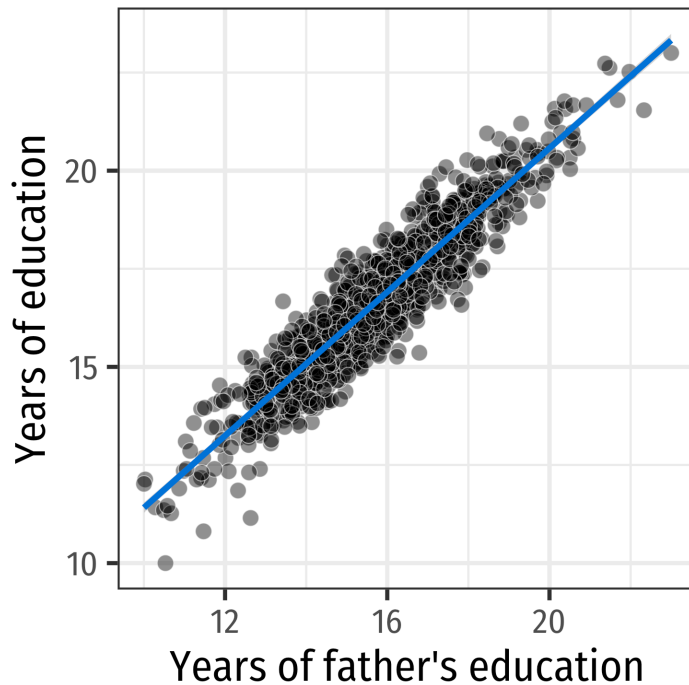
$$\beta_0 + \beta_1 \text{Education}_i^{\text{exog.}} + \underbrace{\beta_1 \text{Education}_i^{\text{endog.}}}_{\omega_i} + \varepsilon_i$$

$$\beta_0 + \beta_1 \text{Education}_i^{\text{exog.}} + \omega_i$$



# Relevancy

## Program ~ instrument



Clear, significant effect = relevant!

```
first_stage <- lm(educ ~ fathereduc, data = father_education)
tidy(first_stage)
```

```
## # A tibble: 2 × 5
##   term      estimate std.error statistic  p.value
##   <chr>      <dbl>    <dbl>    <dbl>    <dbl>
## 1 (Intercept)  2.25      0.172     13.1 3.67e-36
## 2 fathereduc   0.916     0.0108    84.5 0
```

First-stage model F-statistic (<sub>statistic here</sub>) > 104 = strong instrument

```
glance(first_stage)
```

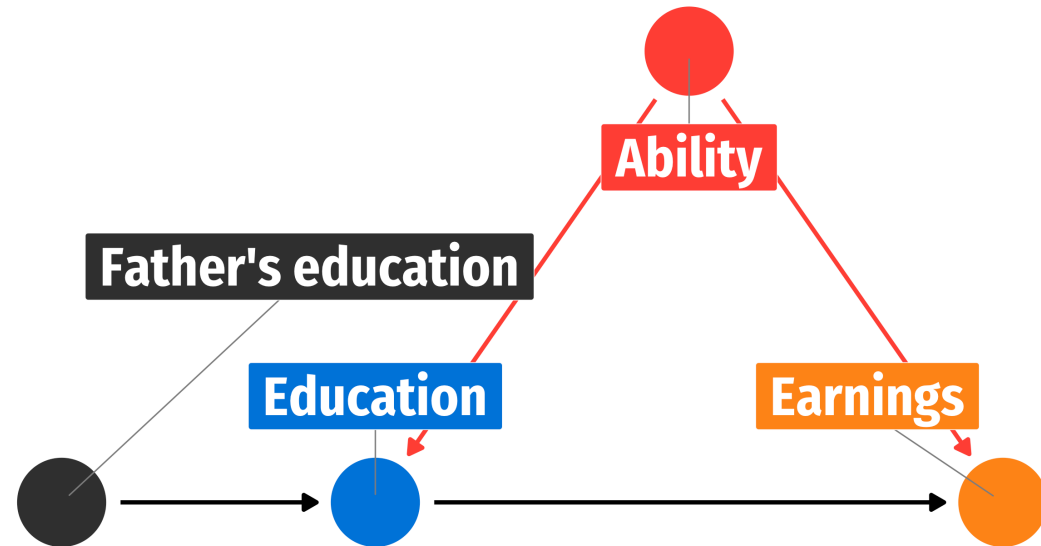
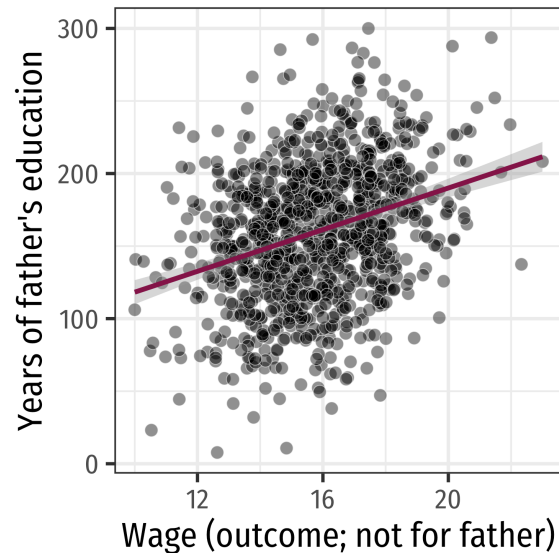
```
## # A tibble: 1 × 12
##   r.squared adj.r.squared sigma statistic p.value    df logLik   AIC   BIC
##   <dbl>      <dbl> <dbl>    <dbl>    <dbl> <dbl> <dbl> <dbl> <dbl>
## 1    0.877      0.877 0.703     7136.      0      1 -1066. 2137. 2152.
## # i 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

# Exclusion

Does it meet exclusion assumption?

Father's education causes your wages *only through* your education?

Any other plausible node between father's education and earnings?



# Exogeneity

**Is assignment to your parents random?**

**Sure.**

**Is your parents' choice to  
gain education random?**

**lolz.**



# Two-stage least squares (2SLS)

Find exogenous part of policy variable based on instrument; use *that* to predict outcome

First stage

$$\widehat{\text{Education}}_i = \gamma_0 + \gamma_1 \text{Father's education}_i + v_i$$

Second stage

$$\text{Earnings}_i = \beta_0 + \beta_1 \widehat{\text{Education}}_i + \varepsilon_i$$

"Education hat": fitted/predicted values;  
exogenous part of education

# Stage 1: Policy ~ instrument

```
first_stage <- lm(educ ~ fathereduc, data = father_education)
tidy(first_stage)
```

```
## # A tibble: 2 × 5
##   term          estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)    2.25      0.172      13.1 3.67e-36
## 2 fathereduc    0.916     0.0108     84.5 0
```

# Stage 1: Check instrument strength

**Model's F-statistic** (statistic here) **should be > 104**  
**(though most books say > 10)**

```
glance(first_stage)
```

```
## # A tibble: 1 × 5
##   r.squared adj.r.squared sigma statistic p.value
##   <dbl>      <dbl> <dbl>      <dbl>      <dbl>
## 1    0.877    0.877 0.703    7136.         0
```

# Stage 1: Use first stage to predict policy

$$\widehat{\text{Education}}_i = 2.251 + (0.916 \times \text{Father's education}_i) + v_i$$

```
data_with_predictions <- augment_columns(first_stage, data = father_education) |>
  rename(educ_hat = .fitted)
head(data_with_predictions)
```

```
## # A tibble: 6 × 5
##   wage educ ability fathereduc educ_hat
##   <dbl> <dbl>   <dbl>      <dbl>   <dbl>
## 1  180.  18.5   408.      17.2    18.0
## 2  100.  16.2   310.      15.5    16.4
## 3  125.  18.2   303.      17.7    18.4
## 4  178.  16.6   342.      15.6    16.5
## 5  265.  17.3   534.      14.7    15.8
## 6  187.  17.5   409.      16.0    16.9
```

$$\text{educ\_hat} = 2.251 + (0.916 \times 17.2) = 18.0$$

$$\text{educ\_hat} = 2.251 + (0.916 \times 15.5) = 16.4$$

# Stage 2: Outcome ~ predicted policy

```
second_stage <- lm(wage ~ educ_hat,  
                   data = data_with_predictions)
```

```
tidy(second_stage)
```

```
## # A tibble: 2 × 5  
##   term          estimate std.error statistic  p.value  
##   <chr>          <dbl>      <dbl>      <dbl>    <dbl>  
## 1 (Intercept)    28.8      12.7        2.27 2.32e- 2  
## 2 educ_hat       7.83      0.755      10.4 5.10e-24
```

|   | Unadjusted | Forbidden  | 2SLS IV  |
|---|------------|------------|----------|
| (Intercept)                                       | -59.378*** | -85.571*** | 28.819*  |
|   | (10.376)   | (7.198)    | (12.672) |
| educ  | 13.124***  | 7.767***   |          |
|   | (0.618)    | (0.456)    |          |
| ability   |            | 0.344***   |          |
|   |            | (0.010)    |          |
| educ_hat  |            |            | 7.835*** |
|   |            |            | (0.755)  |
| Num.Obs.  | 1000       | 1000       | 1000     |
| R2  | 0.311      | 0.673      | 0.097    |
| RMSE  | 39.13      | 26.97      | 44.80    |
| + p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001 |            |            |          |

**Unadjusted  
is wrong!**

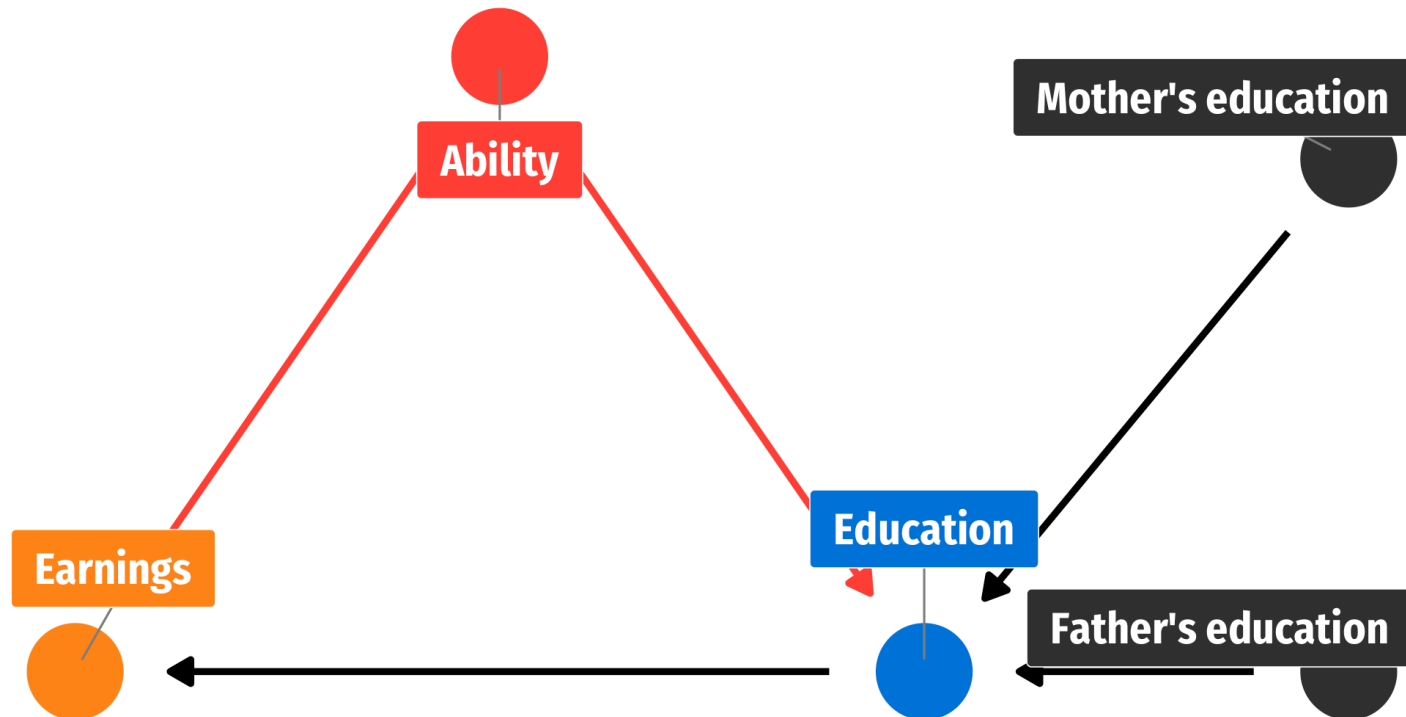
**Forbidden is right,  
but not actually  
measurable!**

**2SLS is close  
*and* measurable!**

**One year of education  
causes hourly wage to  
increase by \$7.84**

# Multiple instruments

You can use multiple instruments to explain more of the endogeneity in the policy node



# Multiple instruments

$$\widehat{\text{Education}}_i = \gamma_0 + \gamma_1 \text{Father's education}_i + \gamma_2 \text{Mother's education}_i + v_i$$

$$\text{Earnings}_i = \beta_0 + \beta_1 \widehat{\text{Education}}_i + \varepsilon_i$$



# Other control variables

You can use control variables too!

For mathy reasons,  
all exogenous controls need to go in both stages

$$\widehat{\text{Education}}_i = \gamma_0 + \gamma_1 \text{Father's education}_i + \gamma_2 \text{Mother's education}_i + \gamma_3 \text{SES}_i + \gamma_4 \text{State}_i + \gamma_5 \text{Year}_i + v_i$$

$$\text{Earnings}_i = \beta_0 + \beta_1 \widehat{\text{Education}}_i + \beta_2 \text{SES}_i + \beta_3 \text{State}_i + \beta_4 \text{Year}_i + \varepsilon_i$$

# Faster, more accurate ways to run 2SLS

Running the first stage, calculating policy-hat, then running second stage is neat, but time consuming!

```
first_stage <- lm(educ ~ fathereduc, data = father_education)

data_with_predictions <- augment_columns(first_stage, data = father_education) |>
  rename(educ_hat = .fitted)

second_stage <- lm(wage ~ educ_hat, data = data_with_predictions)
```

Your standard errors will be wrong unless you adjust them with fancy math by hand

**Use R packages that do all that work for you instead!**

# Faster, more accurate ways to run 2SLS

`ivreg()` from the **ivreg** package

Outcome ~ 2nd stage stuff | 1st stage stuff

```
library(ivreg)
model_ivreg <- ivreg(wage ~ educ | fathereduc,
                     data = father_education)
tidy(model_ivreg)
```

```
## # A tibble: 2 × 5
##   term      estimate std.error statistic  p.value
##   <chr>      <dbl>    <dbl>    <dbl>    <dbl>
## 1 (Intercept)  28.8      11.5      2.51 1.21e- 2
## 2 educ        7.83     0.683    11.5 1.13e-28
```

```
summary(model_ivreg)
```

```
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  28.8187    11.4679   2.513   0.0121 *
## educ         7.8349     0.6834  11.465  <2e-16 ***
##
## Diagnostic tests:
##              df1 df2 statistic p-value
## Weak instruments    1 998      7136 <2e-16 ***
## Wu-Hausman          1 997      1102 <2e-16 ***
```

# Faster, more accurate ways to run 2SLS

`iv_robust()` from the **estimatr** package

Outcome ~ 2nd stage stuff | 1st stage stuff

```
library(estimatr)
model_iv_robust <- iv_robust(wage ~ educ | fathereduc,
                             data = father_education)
tidy(model_iv_robust)
```

```
##           term  estimate  std.error statistic      p.value  conf.low  conf.high
## 1 (Intercept) 28.818695 11.1645893   2.581259 9.985789e-03  6.909932 50.727459
## 2      educ    7.834935  0.6635423 11.807739 3.281862e-30  6.532837  9.137033
##    df outcome
## 1 998   wage
## 2 998   wage
```

(See also `lfe()` from the **felm** package for IV with fancy fixed effects)

|   | Unadjusted | Forbidden  | &nbsp;2SLS IV (by hand) | &nbsp;2SLS IV (ivreg()) | &nbsp;2SLS IV (iv_robust()) |
|---|------------|------------|-------------------------|-------------------------|-----------------------------|
| (Intercept)                                       | -59.378*** | -85.571*** | 28.819*                 | 28.819*                 | 28.819**                    |
|   | (10.376)   | (7.198)    | (12.672)                | (11.468)                | (11.165)                    |
| educ  | 13.124***  | 7.767***   |                         | 7.835***                | 7.835***                    |
|   | (0.618)    | (0.456)    |                         | (0.683)                 | (0.664)                     |
| ability   |            | 0.344***   |                         |                         |                             |
|   |            | (0.010)    |                         |                         |                             |
| educ_hat  |            |            | 7.835***                |                         |                             |
|   |            |            | (0.755)                 |                         |                             |
| Num.Obs.  | 1000       | 1000       | 1000                    | 1000                    | 1000                        |
| R2  | 0.311      | 0.673      | 0.097                   | 0.261                   | 0.261                       |
| R2 Adj.   | 0.311      | 0.672      | 0.096                   | 0.260                   | 0.260                       |
| + p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001 |            |            |                         |                         |                             |

# General IV process

## 1: Is the instrument relevant?

Instrument correlated with policy/program; F-statistic in 1st stage  $> 104$

## 2: Does the instrument meet exclusion assumption?

Instrument causes outcome *only through* policy/program. **Good luck.**

## 3: Is the instrument exogenous?

No arrows going into instrument node in DAG

## 4: 2-stage least squares (2SLS)

`program ~ instrument; outcome ~ program_hat` **OR** `iv_robust()`